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LETTER TO THE EDITOR

Fallacies in the understanding of the quenching of the Hall effect: II

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Abstract. A threshold magnetic field, B_{thres}, at which the skipping orbits begin to be formed, is obtained and an inequality between B_{thres} and B_{crit}, up to which the Hall effect remains quenched, is produced. It is shown that B_{thres} can at best be the lower bound for B_{crit}. This renders untenable suggestions that the quenching phenomenon could be related to the skipping orbits. The quenching, thus, has neither a classical nor a semi-classical origin—it is a quantum mechanical phenomenon with a mechanism suggested by one of the authors.

In a recent letter [1] one of us made an attempt to explain the phenomenon of quenching of the Hall effect (HE) [2, 3] by treating the electron as a classical particle impinging freely on the two edges of a quasi-one-dimensional (1D) system. After introducing the possibility of *specular* reflection from the edges we found that the HE could be quenched in the extreme situation where *all* the reflections, at all angles, were specular with probability one. This was argued to be too strict a condition to be met in an actual experiment.

In this Letter we attempt to explain the above-mentioned 'quenching' phenomenon using a semi-classical approach, based on the idea of conduction along the 'skipping orbits' at the edges first proposed by Lifshitz and Kosevich (LK) [4]. In the HE geometry if the magnetic field $\boldsymbol{B}$ is so weak that the size of the smallest Landau orbit is larger than the width of the system, then according to LK the trajectory of an electron can take the shape of a series of connected segments of Landau orbits that are truncated due to collisions with the edge and specular reflections at the edge (shown in figure 1). The area of each segment of the orbit is to be quantised according to the Bohr–Sommerfeld quantisation rule:

$$\int_{y_1}^{y_2} p\,\mathrm{d}y \equiv B \times (\text{area of a segment}) = (n+\gamma)\frac{hc}{e} \simeq n\frac{hc}{e} \tag{1}$$

$n = 1, 2, 3$ etc and $0 < \gamma < 1$; γ varies slowly with energy and we can ignore it for convenience. Thus a segment encloses an integral number of flux quanta. We will study here whether the highly restricted geometry of the system that exhibits the quenching of the HE imposes any constraint on the formation of the skipping orbits and, furthermore, whether there is any connection between the quenching and the formation/non-formation of the skipping orbits. Beenakker and van Houten (BH) [5] have suggested that below a threshold magnetic field, B_{thres}, skipping orbits are not formed and this leads to the quenching of the Hall voltage. In [1] we have already shown that if the reason for the non-formation of the skipping orbits is that the electrons in the quasi-1D system are frequently colliding against the two edges as is contended by BH [5], then this *cannot* lead to the quenching. So we will first analyse the question of formation/non-formation of skipping orbits.

Note that the semi-classical analysis of LK (see [6] for a detailed discussion) only appears to indicate that so long as a segment of a Landau orbit falling within a system can enclose *at least* one quantum of flux, a skipping orbit should be formed. Indeed, at very small B, the depth, d, penetrated into the system by a skipping orbit (see figure 1)

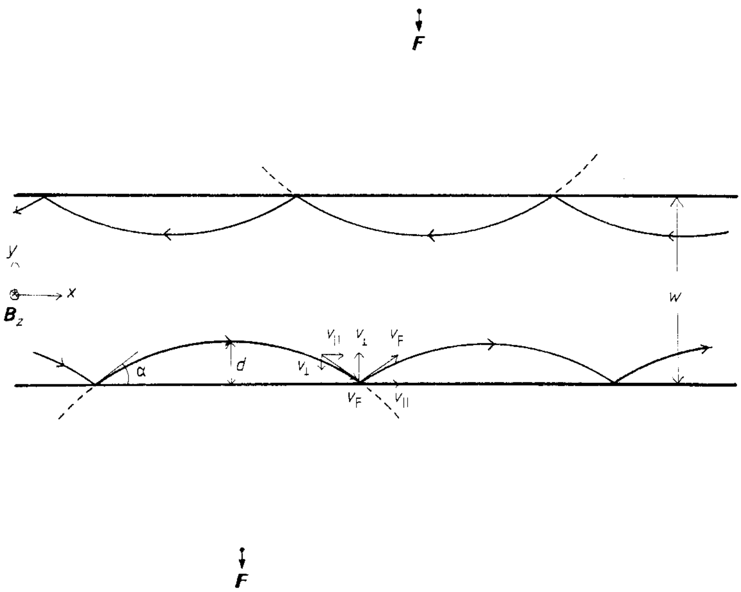

Figure 1. Trajectories of two electrons skipping in opposite directions over the two edges of a quasi-1D device of width w. The depth d penetrated into the system can take several discrete values. The specular reflection reverses the direction of $v_{\perp}$. Note that the Lorentz force $\boldsymbol{F}$ on both the electrons acts in the same direction ($\boldsymbol{F} = e[E_x + (1/c)\boldsymbol{v}_F \times \boldsymbol{B}_z]$).

will be large and also the length of the orbit will have to be very large in order to enclose a flux quantum that occupies a large area. But if d is less than the width of the system and there is long-range phase coherence spanning at least the entire length of the system (to preclude scrambling of the phase as the orbit is traversed), the skipping orbits should be formed. If we consider a typical quasi-1D sample used in quenching experiments—100 nm $\times$ 3 μm—we find that for B as low as 0.05 T the system can contain three to four quanta of flux (each requiring about 0.8×10^{-13} m^2 to spread). Thus only if B is so small that one flux quantum requires more than, say, 2×10^{-13} m^2 of area do we expect no skipping orbit to be formed. So, B_{thres} can only be vanishingly small, whereas the experiments [2, 3] indicate that the HE remain quenched for B as high as 0.1 T. We can, therefore, conclude that the experimental value of B_{crit}, below which the HE remains quenched, is much greater than B_{thres} and that in between B_{thres} and B_{crit} (i.e., for $B_{\text{thres}} < B < B_{\text{crit}}$) the skipping orbits exist but the HE remains quenched.

However, there is a flaw in the above estimation of B_{thres} stemming from the fact that the system of interest is extremely narrow—quasi-1D. The uncertainty in the energy

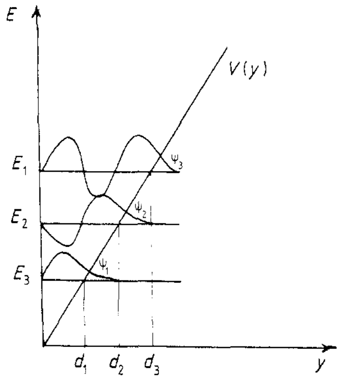

Figure 2. A schematic plot of the energy levels and wavefunctions of the edge-bound states. The corresponding skipping trajectories penetrate distances d_1, d_2, d_3 etc—a discrete set of values. ($V(y) = (e/c)v_F B_y$.)

measurement in a quasi-1D system of width w is given by

$$\Delta\varepsilon \simeq \hbar^2/mw^2 \tag{2}$$

which is a few orders of magnitude *larger* than the quantised stationary states granularity ΔE_n, for the electrons making a periodic movement at velocity v_F along the skipping trajectories, given by [6]

$$\Delta E_n = (e/c)\, v_F B d_n. \tag{3}$$

where the d_n are discrete values of the orbit penetration obtained in accordance with rule (1)†. (Expression (2) is obtained by taking the uncertainty in position in the y direction to be $\simeq w$ and by noting that the uncertainty in the measurement of the y momentum ($\simeq \hbar/w$) is comparable to the y momentum itself‡.) So, because it is less than the uncertainty (2), the quantisation (3) cannot be observed. Therefore skipping trajectories cannot be formed unless ΔE_n becomes comparable to $\Delta\varepsilon$. Using this criterion

$$\Delta\varepsilon = \Delta E_n \tag{4}$$

we can obtain a better estimate of B_{thres},

$$B_{\text{thres}} = (\hbar c/e)/k_F w^2 d_1 \tag{5}$$

for, say, $n = 1$. Since d_1 will be of the order of w, B_{thres} should behave as $k_F^{-1} w^{-3}$. This result is the same as that obtained by BH [5] (except for a factor of $(2\pi)^{-1}$) although the underlying argument here is very different from theirs. Before we begin to investigate whether the normal HE will be set up above B_{thres} we would like to note some useful information related to condition (4) and expression (5).

Our system will start behaving two-dimensionally, magnetically either (i) when $\Delta\varepsilon$ becomes comparable to the Landau quantisation or (ii) when the magnetic length $\sqrt{\hbar c/eB}$ becomes comparable to the width w of the system, as pointed out in [3]. The

† The energy level system (3) may be considered as a series of allowed states for electrons in a potential well formed on one side by the infinite potential step at $y = 0$ and on the other by the potential $V(y) = (e/c)v_F B_y$. In other words the Lorentz force ($v_F \times B$) acting on an electron is replaced by a force always directed along the normal to the edge of the system. Solving the Schrödinger equation for this problem gives the energy levels and the wavefunctions shown in figure 2 [7], the energy levels being the same as given by (3). Note that as B increases, $V(y)$ becomes steeper and ΔE_n increases, i.e. the orbits become smaller and the skipping trajectories are drawn closer to the edges.

‡ k_y takes the values $2\pi i/w$ ($i = 1, 2, \ldots$) i.e., $p_y = hi/w$; at the same time $\Delta p_y \Delta y \simeq \hbar$, i.e. $\Delta p_y \simeq \hbar/\Delta y$ or $\Delta p_y \sim \hbar/w$.

criteria (i) and (ii) are essentially the same and indeed lead to the same answer for the *marginal* magnetic field, B_m, that marks the quasi-1D–2D transition (magnetically speaking)†. The value of B_m so obtained is

$$B_m = (\hbar c/e)/w^2. \tag{6}$$

Clearly B_{thres} and B_m are identical because at this point ΔE_1 becomes the same as the Landau quantisation, and since at this point B_{thres} and B_m exceed the energy uncertainty $\Delta\varepsilon$, the energy quantisation first becomes apparent. Thus we learn that the first skipping trajectory is formed only when the system begins to behave two-dimensionally from the magnetic point of view, and when the trajectory is able to penetrate into the system a depth $d_1 = k_F^{-1}$. In a system about 100 nm wide d_1 will be about 20 nm. We also learn that the B_{thres} depends on the system width as w^2 and *not* as w^{-3}, as contended by BH [5].

After gathering all this information about B_{thres} we can now move on to our main concern of examining the relationship between the quenching and the skipping orbits—should the formation of skipping orbits above B_{thres} automatically lead to the normal HE?

Suppose there is some mechanism that quenches the Hall voltage for B less than *some* B_{crit} (let us, for the time being, not worry about the mechanism). Then in the presence of an electric field in the x-direction (in the plane of the system and normal to $\boldsymbol{B}$) the electrons of mass m will experience a force in the y direction which will obey the following relation only for $B \simeq B_{crit}$ (for which $E_y = 0$):

$$m\,\mathrm{d}v_y/\mathrm{d}t = (e/c)Bv_x \tag{7}$$

or

$$Bv_x = (\hbar c/e)\,\delta k_y/\delta t. \tag{8}$$

In (7) we have taken the collision time τ to be very large (a valid approximation in the ballistic transport regime), so τ^{-1} is ignored. $E_y = 0$ implies that there is no net charge accumulation on one of the edges, i.e. $\boldsymbol{B}$ is not able to produce a deflection in the electron trajectories. Equation (7) (and so (8)) holds when $\boldsymbol{B}$ is just able to produce the desired deflection. Just beyond this point of time one edge of the system becomes more negative than the other and E_y will be set up to oppose the deflection of the charge. In a quasi-1D system of width w, δk_y will have to be of the order of $1/w$ ($\Delta k_y \Delta y \simeq 1$, or $\Delta k_y \simeq 1/\Delta y$, i.e. $\Delta k_y \simeq 1/w$). Also, in the ballistic transport regime [3] of interest here the mean free path l is quite large, so δt, the time interval over which k_y changes by $1/w$, will be *less than* l/v_x. Then B_{crit}, the smallest magnetic field that can produce a change in k_y of the order of $1/w$ and thereby change the energy by at least $\Delta\varepsilon$, will obey the following inequality:

$$B_{crit}\, v_x > (\hbar c/e)\, v_x/wl. \tag{9}$$

At B_{crit} the electron trajectories experience a net deflection in the y direction, i.e. the mechanism that prevented the deflection for $B < B_{crit}$ is overcome. Now one edge becomes more negative than the other and E_y is established. This critical magnetic field is found to obey the inequality

$$B_{crit} > (\hbar c/e)/wl. \tag{10}$$

† Note that in [3] $w = 2\sqrt{\hbar c/eB}$ is taken as the criterion for two-dimensionality, whereas we drop the factor of two to get the result (6). Our argument for this is that for $B > B_m$, since the Landau quantisation $\hbar w_c$ becomes larger than $\Delta\varepsilon$, the relevant length scale at B_m, which marks the onset of Landau quantisation, is $\sqrt{\hbar c/eB}$ and not double this.

For $l \geqslant w$ we get

$$B_{\mathrm{crit}} \gtrsim (\hbar c/e)/w^2. \tag{11}$$

Comparing this with (6) (and recalling that $B_{\mathrm{m}} \equiv B_{\mathrm{thres}}$), we find that

$$B_{\mathrm{crit}} \gtrsim B_{\mathrm{thres}} \tag{12}$$

i.e. B_{thres}, at best, forms the lower bound for the onset of the normal HE so long as we are in the ballistic transport regime. In general, B_{crit} is greater than B_{thres}, i.e. *the HE can remain quenched in spite of the formation of the skipping orbits* contrary to the suggestion of BH [5]. To show conclusively that the quenching of the HE should not be linked with the skipping orbits in any way we show in the following how in the systems of interest the skipping orbits remain unaffected by the presence of the Lorentz force and, therefore, do not interfere with the mechanism (whatever it is) responsible for the quenching.

The dragging force acts on the centres of the arcs of the segments and is normal to both the electric and magnetic fields. Since the sense of movement in the trajectories at the two edges is the same (say clockwise as in figure 1), the dragging force on both these trajectories will act in the same direction. In the present situation where because of the small width of the system more than one trajectory, the d_1-trajectory, may not be accommodated on either edge, the trajectory along the lower edge cannot be shrunk closer to the edge because it is already enclosing the minimum of one flux quantum. The trajectory along the upper edge can possibly move downwards if there is space available but since the rule (1) is to be obeyed, this movement will have to be at the expense of the deformation of the orbit. The depth d and the angle of reflection α will increase. This will have the serious consequence of altering the potential $V(y)$ and increasing the energy ΔE_1. For this reason the downward movement of the upper trajectory may be forbidden. The Bohr–Sommerfeld quantisation, thus, ties the skipping trajectories to the edges.

Another possibility is that the electrons near the upper edge moving in the d_1-trajectory may be pulled downwards into the d_2-trajectory and, to keep the energy of the system unchanged, the electrons near the lower edge may be pulled up into the d_2-trajectory *against* the Lorentz force. Although this strengthens the argument for the quenching of the HE, such a possibility has to be ruled out for the reasons given below.

The skipping orbits can be viewed as vortices of circulating current (in a mathematical sense) because (i) they trap an integral number of flux quanta, and (ii) the Bohr–Sommerfeld phase of a periodic trajectory has been recognised as being Berry's geometrical phase [8] implying that the periodic skipping orbits are equivalent to a cyclic movement *similar* to the one that gives rise to the Aharonov–Bohm phase [9, 10]. A transition from the d_1-trajectory into the d_2-trajectory would amount to the dilation of a vortex initially encircling one flux quantum until it is big enough to enclose two flux quanta. This is disallowed by Kelvin's theorem (see, e.g., [11]) according to which the strength of a vortex (the number of flux quanta in this case) must be a constant. If the field increases (increasing the magnitude of curl $\boldsymbol{v}$ in the case of fluid mechanics) then the cross section of the filament or the area of the vortex must reduce to keep the vortex strength (= area times curl $\boldsymbol{v}$) constant. We see that the change in field strength cannot induce a transition between vortices of different strengths. In the present case, by varying $\boldsymbol{B}$ we only change d_1 and d_2 etc, i.e. the level separation E_n changes but this cannot induce a transition between two levels E_n. A transition can be produced by scattering by, say, impurities.

Having settled the question of the role of skipping orbits as regards the quenching phenomenon, which was our main concern in this Letter, we can look into a possible mechanism for the quenching ([12]; see also [13]) to check (i) how the skipping orbits do not interfere with it, and (ii) whether the value of B_{crit} obtained for it satisfies the inequality (12).

In the situation where the electron wavefunction spreads over the entire width of the system and the phase coherence exists over length scales larger than w (manifested in the above in l being $\gg w$), one of us proposed a mechanism for the quenching of the Hall voltage based on a subtle idea involving 'phase-driven current' [12, 13]. The critical magnetic field, B_{crit}, for the onset of the normal HE was found to be

$$B_{\mathrm{crit}} = (hc/e)/4w^2 \tag{13}$$

Thus,

$$B_{\mathrm{crit}} = (\pi/2)B_{\mathrm{thres}} \tag{14}$$

which satisfies the inequality (12) as well as yielding excellent agreement with experiments [2, 3].

While in the normal HE E_y is set up to stop J_y, the transverse current due to the Lorentz force, from flowing, in this mechanism [12, 13] the novel proposition is that a phase-driven current $\bar{J}_y$ is set up to counter J_y. Current J_y is thus cancelled and the creation of E_y is suppressed at the outset. As soon as $\boldsymbol{B}$ is turned on, even the slightest current J_y created as a result of it must be countered by an equal and opposite current $\bar{J}_y$ in order to prevent any charge build up on an edge that can, in turn, give rise to a field component E_y. This is exactly the opposite of what happens in the normal HE. Within the framework of the mechanism of [12, 13] the phenomenon of quenching can be termed as an *anti-Hall Effect.*

Turning to the question (i) above we note that apart from those conduction electrons that meet the conditions for specular reflection together with the quantisation rule (1) and thus have their movement quantised by getting locked into the skipping orbits, the movement of all others remains unquantised. We have seen how the former remain unaffected by the dragging force exerted jointly by the electric and magnetic fields. The latter, i.e. the electrons with unquantised movement, are subjected to the mechanism of [12, 13] which aligns their movement in the x direction in the absence of E_y. Thus the presence of the skipping orbits does not interfere with the mechanism of the quenching for $B_{\mathrm{thres}} < B < B_{\mathrm{crit}}$.

Finally, we will touch upon another feature reported in the experiments [2, 3] although it is not related to the main subject of this paper. The experiments show a steep rise in the Hall voltage just above B_{crit}—an attempt to 'catch up' for the lost classical variation of E_y with $\boldsymbol{B}$. We do not offer an explanation for this. We only make a deduction that may prove to be helpful in finding an explanation, should it be necessary. For $B > B_{\mathrm{crit}}$, the field E_y develops and so (7) takes the form

$$m\, \mathrm{d}v_y/\mathrm{d}t = -e[E_y - (1/c)Bv_x]. \tag{7$'$}$$

In the steady state (i.e. where $\dot{v}_y = 0$), the steeper (than classical) rise of E_y indicates that v_x increases as B increases contrary to the typical classical situation where v_x remains independent of B. Since in the experiments J_x is maintained constant, increasing v_x indicates a decrease in n, the number of charge carriers. This implies an increase in localisation. This enhanced localisation above B_{crit} is also indicated in the experiments by positive magnetoresistance (ρ_{xx}) for $B > B_{\mathrm{crit}}$. All this is very similar to what happens in the transition region between the ith plateau and the $(i-1)$th plateau in the quantum Hall regime. We analyse this in detail in a forthcoming paper.

In summary, we point out that for the onset of the HE a deflection of the current-carrying electrons by the Lorentz force is necessary, *not* just the formation of skipping orbits along the edges. In a quasi-1D system we show that the former occurs at a critical magnetic field $\boldsymbol{B}_{\text{crit}}$ that is in general greater in magnitude than the threshold magnetic field $\boldsymbol{B}_{\text{thres}}$ at which the skipping orbits are first formed. For $B < B_{\text{crit}}$ the system does not obey the equation of motion (7) (or (7′)). The reason for this, according to the theory of [12, 13], is that the phase rigidity prohibits the deflection from occurring under the Lorentz force. The net force, therefore, has to be strong enough to overcome the effect of phase rigidity. Also, this has been shown [12, 13] to occur when B becomes greater than B_{crit} given by (13).

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